Mimetic discretization of two-dimensional Darcy convection

B. Karasözen a,*, V.G. Tsybulin b

a Department of Mathematics and Institute of Applied Mathematics, Middle East Technical University, Turkey
b Department of Mathematics and Mechanics, Rostov State University, Russia

Received 28 April 2004; accepted 30 December 2004
Available online 11 March 2005

Abstract

We consider discretization of the planar convection of the incompressible fluid in a porous medium filling rectangular enclosure. This problem belongs to the class of cosymmetric systems and admits an existence of a continuous family of steady states in the phase space. Mimetic finite-difference schemes for the primitive variables equation are developed. The connection of a derived staggered discretization with a finite-difference approach based on the stream function and temperature equations is established. Computations of continuous cosymmetric families of steady states are presented for the case of uniform and nonuniform grids.

© 2005 Elsevier B.V. All rights reserved.

PACS: 02.30.Jr; 02.70.Bf; 47.55.Mh

Keywords: Convection; Porous medium; Darcy law; Cosymmetry; Finite-difference method; Staggered grid; Family of equilibria

Introduction

Convection of incompressible fluid in a porous medium is the subject of many theoretical and numerical works [1]. Usually, the problem consists of a finite number of regimes (convective patterns) separately distributed in the phase space. An exciting example with an infinite number of steady states was observed for the planar problem of incompressible fluid convection in a porous medium based on the Darcy law [2]. This case of the appearance of a continuous family of steady states was explained by the cosymmetry theory [3,4]. Computations for the given problem formulated as a system of two unknowns—stream function and temperature—were performed using the Galerkin method [5,6], the finite-difference scheme [7,8] and a combination of spectral and finite-difference approaches [9–11]. The mimetic [12,13] approximation of underlying system is a crucial point for the computa-
tion of the family of steady states. It was found that the loss of cosymmetry property in the finite-dimensional approximation destroys the family of steady states and leads to a number of isolated stationary regimes [7,9].

In this work we deal with the two-dimensional problem of natural convection in a porous medium and develop a finite-difference scheme for the system formulated in primitive variables (velocities, pressure and temperature). The discretization on staggered grids is presented and the algorithm for the computation of the family of steady states is briefly discussed. Numerical results on the branching off of the family of steady states from the state of rest are presented.

1. Problem formulation

1.1. Primitive variables equations

We consider a rectangular region (enclosure) \( D = [0, a] \times [0, b] \) filled by a porous medium saturated by an incompressible fluid which is heated below. The system of equations consists of the momentum equation based on the Darcy law

\[
\nabla p + \vec{V} - \theta \vec{y} = 0,
\]

(1)

the continuity equation

\[
\nabla \cdot \vec{V} = 0,
\]

(2)

and the energy equation

\[
\dot{\theta} = \Delta \theta - \lambda \vec{V} \cdot \vec{y} - (\vec{V} \cdot \nabla) \theta.
\]

(3)

Here \( \vec{V} = (u(x, y, t), v(x, y, t)) \) is the velocity vector, \( \vec{y} = (0, -1) \) is the gravity direction, \( p(x, y, t) \) is the pressure, \( \theta(x, y, t) \) is the deviation of the temperature from a linear (in \( y \)) profile, the dot accent denotes differentiation in time \( t \).

The Rayleigh number is defined as \( \lambda = \alpha g T K l / \chi \mu \), where \( \alpha \) is the thermal expansion coefficient, \( g \) is the gravity acceleration, \( \mu \) is the kinematic viscosity, \( \chi \) is the thermal diffusivity of the fluid, \( K \) is the permeability coefficient and \( l \) is the length parameter. We suppose that the temperature at the boundary is given by a linear function on the vertical coordinate \( y \). So, its deviation \( \theta \) is equal to zero at the boundary \( \partial D \) as well as the normal component of the velocity:

\[
\theta = 0, \quad \vec{V} \cdot \vec{n} = 0, \quad (x, y) \in \partial D.
\]

(4)

The initial condition is formulated only for the temperature

\[
\theta(x, y, 0) = \theta_0(x, y)
\]

(5)

because the other unknowns can be found as solutions of system (1), (2), (4).

It is simple to check that Eqs. (1)–(4) are invariant with respect to the discrete symmetries

\[
R_x : \{x, y, u, v, p, \theta\} \mapsto \{a - x, y, -u, v, p, \theta\},
\]

(6)

\[
R_y : \{x, y, u, v, p, \theta\} \mapsto \{x, b - y, u, -v, p, -\theta\}.
\]

(7)

This implies the existence of the solutions following from the given ones by an appropriate transformation of velocity, pressure and deviation of the temperature.
1.2. System for the temperature and stream function

The continuity equation (2) is fulfilled automatically when the stream function $\psi$ is introduced

$$u = -\psi_y, \quad v = \psi_x. \tag{8}$$

Then the underlying system can be transformed to the another form. After application of the $\text{curl}$-operator to (1) we deduce

$$0 = \Delta \psi - \theta_x \equiv G, \tag{9}$$

and using (8) we obtain from (3)

$$\dot{\theta} = \Delta \theta + \lambda \psi_x - J(\psi, \theta) \equiv F, \quad J(\psi, \theta) = \psi_x \theta_y - \psi_y \theta_x. \tag{10}$$

The boundary conditions for the system (9), (10) follow from (4):

$$\psi = \theta = 0, \quad (x, y) \in \partial D. \tag{11}$$

Eqs. (9)–(11) impose the equilibrium $\theta = \psi = 0$ (quiescent state), being stable if $\lambda < \lambda_{11}$, where $\lambda_{nm} = (2\pi n/a)^2 + (2\pi m/b)^2$ ($m, n \in \mathbb{Z}$) are the eigenvalues for the corresponding spectral problem. It was shown in [4] that the first critical value $\lambda_{11}$ has multiplicity two for any domain $D$. As a result, the continuous family of steady states appears [2,3]. Then whenever $\lambda$ passes through $\lambda_{nm}$ ($n + m > 2$), this produces a new family with unstable equilibria.

System (9)–(11) possesses a cosymmetry [3]: a vector-function $(\theta, -\psi)$ is being orthogonal to the right-hand side of (9), (10) in $L_2$. Then, we come to the cosymmetry condition in the following form

$$\int_D (F\psi - G\theta) \, dx \, dy = \int_D \left[ \Delta \theta \psi - \Delta \psi \theta + \lambda \psi_x \psi + \theta_x \theta + J(\theta, \psi) \psi \right] \, dx \, dy = 0. \tag{12}$$

This can be checked directly using integration by parts and Green’s formulae.

2. Spatial discretization

To approximate equations (9)–(11) it is natural to impose a discrete version of the cosymmetry equality. In [7] the regular uniform mesh was used and both stream function and temperature were defined at common nodes. The Jacobian approximation was based on the Arakawa scheme [14] and a number of one-parameter families of steady states were computed. It was also found that a violation of the cosymmetry property leaded to a degeneration of the family. As a result, there exist only a finite number of isolated equilibria branching from zero equilibrium when $\lambda$ passes the first critical value. In [8] the application of staggered grids was considered for Eqs. (9)–(11), (5), and different nodes were used for the temperature and the stream function. The Jacobian approximation was based on [15].

We apply here a finite-difference Marker and Cell Method (MAC) [16]. Discretization is constructed using four types of nodes: nodes for pressure, temperature, horizontal and vertical components of velocity. Firstly, we define the grid for the temperature $\theta$

$$\omega_\theta = \{(x_i, y_j), 0 = x_0 < x_1 < \cdots < x_{n+1} = a, \quad 0 = y_0 < y_1 < \cdots < y_{m+1} = b \}. $$

Then we introduce the staggered grids along the $x$- and $y$-coordinates

$$x_{i+1/2} = \frac{1}{2}(x_i + x_{i+1}), \quad i = 0, \ldots, n,$$

$$h_i = x_{i+1/2} - x_{i-1/2}, \quad i = 1, \ldots, n,$$
The nonlinear terms in (3) are approximated using averaging and differencing operators on four-points stencil

\[ h_{i+1/2} = x_{i+1} - x_i, \quad i = 0, \ldots, n, \]
\[ y_{j+1/2} = \frac{1}{2}(y_j + y_{j+1}), \quad j = 0, \ldots, m, \]
\[ g_j = y_{j+1/2} - y_{j-1/2}, \quad j = 1, \ldots, m, \]
\[ g_{j+1/2} = y_{j+1} - y_j, \quad j = 0, \ldots, m. \]

Here \( h_{i+1/2}, h_i, g_{j+1/2}, \) and \( g_j \) are the step sizes.

The velocities \( u \) and \( v \) are defined on the grids staggered, respectively, on \( y \) and \( x \),

\[ \omega_u = \{(x_i, y_{j+1/2}), \quad i = 0, \ldots, n + 1, \quad j = 0, \ldots, m\}, \tag{13} \]
\[ \omega_v = \{(x_{i+1/2}, y_j), \quad i = 0, \ldots, n, \quad j = 0, \ldots, m + 1\}. \tag{14} \]

Finally, the pressure \( p \) is defined at the nodes

\[ \omega_p = \{(x_{i+1/2}, y_{j+1/2}), \quad i = 0, \ldots, n, \quad j = 0, \ldots, m\}. \tag{15} \]

We use superscripts and subscripts to denote the nodes on the coordinate \( y \) and \( x \) respectively. So we have

\[ u^{i+1/2}_i = u(x_i, y_{j+1/2}), \quad v^{i+1/2}_i = v(x_{i+1/2}, y_j), \quad p^{i+1/2}_i = p(x_{i+1/2}, y_{j+1/2}), \quad \theta^{i+1/2}_i = \theta(x_i, y_j). \]

### 2.1. Differentiation operators

To approximate (1)–(4) we define a number of discrete analogs of differential operators [17]. Firstly, we introduce the first order difference operators on the two-point stencil

\[ (\delta_x f)_{i+1/2} = \frac{f_{i+1} - f_i}{x_{i+1} - x_i} \approx f_x(x_{i+1/2}, y_{j+1/2}), \tag{16} \]
\[ (\delta_y f)_{i+1/2} = \frac{f_{i+1} - f_i}{y_{j+1} - y_j} \approx f_y(x_{i+1/2}, y_{j+1/2}) \tag{17} \]

and for each coordinate a weighted averaging operator

\[ (\delta^0_x f)_{i+1/2} = \frac{f_{i+1} + f_i}{2(x_{i+1} - x_i)} \approx f_x(x_{i+1/2}, y_{j+1/2}), \tag{18} \]
\[ (\delta^0_y f)_{i+1/2} = \frac{f_{i+1} + f_i}{2(y_{j+1} - y_j)} \approx f_y(x_{i+1/2}, y_{j+1/2}). \tag{19} \]

Formulae (16)–(19) are valid both for integer and half-integer values of \( i \) and \( j \). Then we can derive the operators for another stencils. The first order differencing operators on the three-nodes stencil are defined as follows

\[ (D_x f)_{i}^j = (\delta^0_x f)_{i}^j \approx f_x(x_i, y_j), \quad (D_y f)_{i}^j = (\delta^0_y f)_{i}^j \approx f_y(x_i, y_j), \tag{20} \]

and the discrete analog of the Laplacian on the five-nodes stencil is written as

\[ (\Delta f)_{i}^j = (\delta_x \delta_x f + \delta_y \delta_y f)_{i}^j \approx (\Delta f)_{i}^j. \tag{21} \]

The nonlinear terms in (3) are approximated using averaging and differencing operators on four-points stencil

\[ d_0 = \delta^0_0 \delta^0_0, \quad d_x = \delta^0_0 \delta^0_0, \quad d_y = \delta^0_0 \delta^0_0, \quad d_{xy} = \delta^0_0 \delta^0_0 \delta^0_0 \delta^0_0. \tag{22} \]

We stress that these operators have variative forms for different meshes. For instance, on the mesh \( \omega_p \) we have

\[ (d_0 f)_{i+1/2}^j = \frac{1}{4}(f_{i+1}^j + f_{i+1}^j + f_{i}^j + f_{i+1}^j). \]
\[(d_\varepsilon f^j_{i+\frac{1}{2}}) = \frac{1}{2h_{i+\frac{1}{2}}} \left( f^j_{i+\frac{1}{2}} - f^j_i - f^j_{i+1} \right) \]
\[(d_\eta f^j_{i+\frac{1}{2}}) = \frac{1}{2g_{j+\frac{1}{2}}} \left( f^j_{i+\frac{1}{2}} + f^j_i - f^j_{i+1} \right) \]
and for the nodes belonging to \(\omega_0\)

\[(d_0 f^j_{i+\frac{1}{2}}) = \frac{y_j - y_{j-1/2}}{h_{i+1/2}} \left( (\xi_{i+1/2} - \xi_i) f^j_{i+1/2} + (\xi_i - \xi_{i-1/2}) f^j_{i-1/2} \right) + \frac{y_j - y_{j-1/2}}{h_{i+1/2}} \left( (\xi_{i+1/2} - \xi_i) f^j_{i+1/2} + (\xi_i - \xi_{i-1/2}) f^j_{i-1/2} \right), \]

\[(d_\varepsilon f^j_{i+\frac{1}{2}}) = \frac{y_j - y_{j-1/2}}{h_{i+1/2}} \left( (\xi_{i+1/2} - \xi_i) f^j_{i+1/2} + (\xi_i - \xi_{i-1/2}) f^j_{i-1/2} \right) + \frac{y_j - y_{j-1/2}}{h_{i+1/2}} \left( (\xi_{i+1/2} - \xi_i) f^j_{i+1/2} + (\xi_i - \xi_{i-1/2}) f^j_{i-1/2} \right). \]

2.2. Semi-discretization of the primitive variables problem

Using the introduced operators (16)–(22) we discretize system (1)–(3)

\[0 = (\delta_x u + \delta_y v)^{i+\frac{1}{2}}, \quad i = 0, \ldots, n, \quad j = 0, \ldots, m, \quad \text{(23)}\]
\[0 = (\delta_x u + u^i)^{i+\frac{1}{2}}, \quad i = 0, \ldots, n, \quad j = 1, \ldots, m, \quad \text{(24)}\]
\[0 = (\delta_y v - \frac{\lambda_0}{h_0} v)^{i+\frac{1}{2}}, \quad i = 1, \ldots, n, \quad j = 0, \ldots, m, \quad \text{(25)}\]
\[\dot{\theta}^j_i = (\Delta_\theta \theta + \lambda \delta_{\theta}^0 v - \alpha J_D - (1 - \alpha) J_d)^i, \quad i = 1, \ldots, n, \quad j = 1, \ldots, m. \quad \text{(26)}\]

Here the nonlinear term is constructed using the linear combination of two nonlinear different operators \(J_D\) and \(J_d\) with a free parameter \(\alpha\) as in [17]

\[J_D = D_x (\theta \delta_{\theta}^0 u) + D_y (\theta \delta_{\theta}^0 v), \quad J_d = d_x (d_\theta \theta \delta_{\theta}^0 u) + d_y (d_\theta \theta \delta_{\theta}^0 v). \quad \text{(27)}\]

Both \(J_D\) and \(J_d\) provide second order accuracy for the uniform grid and being asymptotically second order accurate in the case of quasi-uniform mesh. It was shown in [17] that a combination of \(J_D\) and \(J_d\) with \(\alpha = 1/3\) allows to conserve energy and total or mean square vorticity (enstrophy) in the finite-difference approximation of two-dimensional flow and thus being mimetic for the underlying problem.

The grids \(\omega_0, \omega_c\) are introduced in such a way that the nodes for the transversal velocity are located on the wall. It allows us to realize the boundary condition on a rigid wall (4) in a very simple way:

\[u^i_{0+\frac{1}{2}} = 0, \quad j = 0 \div m, \quad \theta^i_0 = 0, \quad j = 0 \div m + 1, \quad \text{(28)}\]
\[u^i_{n+\frac{1}{2}} = 0, \quad j = 0 \div m, \quad \theta^i_{n+1} = 0, \quad j = 0 \div m + 1, \quad \text{(29)}\]
\[v^i_{i+\frac{1}{2}} = 0, \quad i = 0 \div n, \quad \theta^i_{n+1} = 0, \quad i = 0 \div n + 1, \quad \text{(30)}\]
\[v^i_{i+\frac{1}{2}} = 0, \quad i = 1 \div n, \quad \theta^i_0 = 0, \quad i = 1 \div n + 1. \quad \text{(31)}\]

2.3. Transition to the stream function and temperature system

It can be easily checked that the discrete equations (23)–(31) allow us to eliminate the pressure and velocities. Then we formulate the system with the discrete stream function and temperature. After introducing the stream
function $\psi$ at the nodes $\omega_\theta$ and the application of difference operators (16) and (17) we obtain
\begin{equation}
(\delta_y \psi)_{j+1/2} = -u_j^{1/2}, \quad (\delta_x \psi)_{i+1/2} = v_i^{1/2}.
\end{equation}
(32)

We stress that Eq. (23) is automatically fulfilled by (32). Then, we substitute (32) to (24) and (25) and deduce
\begin{equation}
(\Delta_h \psi)_{i,j} - (D_x \theta)_{i,j} = 0.
\end{equation}
(33)

Similarly we find from (26)
\begin{equation}
\dot{\theta}_{i,j} = [(\Delta_h \theta) + \lambda (D_x \psi) + (1 - \alpha) J_D]_{i,j},
\end{equation}
(34)

where
\begin{equation}
J_D = D_x(D_y \psi) - D_y(D_x \psi), \quad J_d = dx \left[ (d_0 \partial d_x \psi) - (d_0 \partial d_y \psi) \right].
\end{equation}
(35)

The resulting scheme (33)–(35) ensures the fulfillment of a discrete analog of cosymmetry property (12) as well as nullification of gyroscopic terms. Thus we can generalize the scheme in [7] to the case of nonuniform rectangular grids using primitive variables for the temperature and stream function. Equations in [7] follow from (33)–(35) for the case of the uniform grid $x_i = ih, y_j = jg$, $h = a/(n+1), g = b/(m+1)$. Then the Jacobian approximation gives the famous Arakawa formula [14] for $h = g$.

### 3. Computation of the family of steady states

For hydrodynamical problems a subdivision of the underlying system is widely used: an equation for the pressure and a subsystem for the velocities and the temperature. To compute the families of steady states we deduce the discrete system only for the temperature. Let us introduce the vectors
\begin{align*}
P &= (p_1^{1/2}, p_2^{1/2}, \ldots, p_{n+1/2}^{m+1/2}), \quad \Theta = (\theta_1^1, \theta_2^1, \ldots, \theta_{n}^m), \\
U &= (u_1^{1/2}, u_2^{1/2}, \ldots, u_{n}^{m+1/2}), \quad V = (v_1^{1/2}, v_3^{1/2}, \ldots, v_{n+1/2}^{m+1/2}),
\end{align*}

and rewrite the discrete equations (23)–(31) in the form
\begin{align}
B_1 U + B_2 V &= 0, \\
B_3 P + U &= 0, \\
B_4 P + V - C_1 \Theta &= 0, \\
\dot{\Theta} &= \lambda \Theta + \lambda C_2 V - \Phi.
\end{align}
(36)–(39)

Here the matrices $B_k, k = 1, \ldots, 4$, correspond to the first order difference operators, the matrices $C_1$ and $C_2$ correspond to the averaging operators and the matrix $\lambda$ is the discrete form of the Laplacian. The nonlinear term is given by $\Phi = \Phi(U, V, \Theta)$. Eqs. (36)–(39) form a system of $4nm + 2(n + m) + 1$ unknowns.

We can express $P, U, V$ via $\Theta$ from (36)–(38) and obtain a system of $n \times m$ ordinary differential equations with unknowns $\Theta$. Substituting (37) and (38) into (36) we deduce
\begin{equation}
BP = B_2 C_1 \Theta, \quad B = (B_1 B_3 + B_2 B_4).
\end{equation}
(40)

Here the matrix $B$ is degenerate with defect 1. Since for an incompressible flow the pressure may differ by a constant we can exclude one component of $P$ and respectively one equation.

To study the convective patterns we apply the direct approach. The system of ordinary differential equations (39) is integrated by the Runge–Kutta method of the fourth order.
To compute a family of steady states we apply the technique based on the cosymmetric version of the implicit function theorem [18] and the algorithm developed in [6,7,19]. The zero equilibrium $U = V = \Theta = 0$ is globally stable for $\lambda < \lambda_{11}$. When $\lambda$ is slightly larger than $\lambda_{11}$, then all points of the family are stable [4]. Starting from the vicinity of unstable zero equilibrium we integrate the ordinary differential equations (39) up to a point $\Theta_0$ close to some stable equilibrium on the family. Then we correct the point $\Theta_0$ by the Newton method. To predict the next point on the family we determine the kernel of the linearization matrix (Jacobian matrix) at the point $\Theta_0$ and use the Adams–Bashford method. This procedure is repeated to obtain the entire family.

4. Numerical results

In [7] we have used aspect ratio $a/b = 2/5$ and in [11] $a/b = 1.6$. Here we present the results of computations for the wide enclosure $a/b = 2/1$. It should be noted that the parametric study of the families of steady states in the case of narrow enclosures with $1 < b/a < 5$ was presented in [11]. We take the rather coarse grids to reveal the difference between uniform grid and nonuniform ones. The nonuniform grids in $x$ direction are only analyzed whenever a grid is uniform in $y$-direction. 24 × 12 and 32 × 16 nodes was used.

Let consider the grids with condensation of the points to the ends ($x = 0, a$) and to the middle ($x = a/2$) of the interval governed by the parameter $q$ (the case $q = 1$ corresponds to the uniform grid)

\[
x_0 = 0, \quad x_{i+1} = x_i + h q^i, \quad i = 0, \ldots, n - 1,
\]

\[
x_{n+1-i} = a - x_i, \quad i = 0, \ldots, n/2, \quad h = \frac{2a(1-q)}{q^{n/2}(1-q) + 2(1-q^{n/2})}.
\]

In Table 1 we present two first critical values of the Rayleigh numbers computed for different values of $q$ at the mesh 24 × 12. Here $\lambda_{11}$ corresponds to the threshold where the trivial equilibrium $\Theta = 0$ becomes unstable and a one-parameter family of stable steady states emerges, and $\lambda_{21}$ is connected with the appearance of a totally unstable family from the trivial equilibrium. One can see that the case of the uniform grid is closest to the exact values.

It should be noted that the mesh 24 × 12 provides an admissible accuracy for the computation of the critical values and convective patterns. Practically the same results were reached on the mesh 40 × 20. In this case however the computation of the continuous family of steady states requires essentially more time.

To present the results of computation on the steady states family we use the Nusselt values [5] defined by

\[
Nu_v = \int_0^b \theta_x \left( \frac{a}{2}, y \right) dy, \quad Nu_b = \int_0^a \theta_y (x, 0) dx,
\]

where $Nu_v$ corresponds to the integral value of the heat flux from the left to the right defined for the centered vertical section of the rectangular domain. The value $Nu_b$ is a combined heat flux through the bottom of the container.

In Fig. 1 we show the families computed for different values of $q$ ($q = 1$ corresponds the uniform grid). One can see that the nonuniform grids deform the family projections (dashed curves).

The temperature and velocity fields for different nonuniform grids are presented in Fig. 2 ($q = 0.8$) and Fig. 3 ($q = 1.2$). For comparison in Fig. 4 the case of a uniform grid is shown. There are four the steady states from the

<table>
<thead>
<tr>
<th>$q$</th>
<th>0.6</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
<th>1.4</th>
<th>Exact</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{11}$</td>
<td>59.78</td>
<td>53.08</td>
<td>50.71</td>
<td>50.52</td>
<td>51.32</td>
<td>53.11</td>
<td>59.18</td>
<td>49.35</td>
</tr>
<tr>
<td>$\lambda_{21}$</td>
<td>137.87</td>
<td>93.13</td>
<td>85.75</td>
<td>83.74</td>
<td>85.40</td>
<td>90.06</td>
<td>106.96</td>
<td>78.96</td>
</tr>
</tbody>
</table>
Fig. 1. Families of steady states for uniform (solid) and nonuniform (dashed) grids, \( \lambda = 60 \); at the left: \( q = 1 \) (curve 1), \( q = 0.9 \), \( q = 0.8 \); at the right: \( q = 1 \) (1), \( q = 1.1 \) (2) and \( q = 1.2 \) (3).

Fig. 2. Temperature and vector field, \( q = 0.8 \).

family in each figure. The temperature and vector field in the top corresponds the extreme right point on the curves in Fig. 1. The other rows follow clockwise: extreme lower, left and upper points. For instance, the state given by the top row in Fig. 4 is characterized by the Nusselt values \( \text{Nu}_h = 37 \) and \( \text{Nu}_v = 0 \).

Because the rather coarse grid one can see the difference in the location of the centers of convective cells vortices. The uniform grid provides better results to this problem. The nonuniform grid may be used if one is interested in an additional resolution to compute the flow in some parts of the enclosure.
The nonuniform grid produces the worst results both for the critical values and patterns of the family of steady states. For a rather nonuniform grid ($q = 0.6$ or $q = 1.4$) we found that the family emerges only at $\lambda \approx 60$. The complete family consists of the states that differ one from another by the number of convective cells and by their location. On the other hand if we are interested to follow a particular regime, then an appropriate distribution of the nodes may be chosen to describe the flow in a more precise way.
5. Conclusion

To compute a continuous family of equilibria one needs to solve repeatedly a nonlinear system being degenerate in the vicinity of the family. This is why discretization is so important for the cosymmetric problem of Darcy convection where a number of such families exist. We have developed the approach based on primitive variables equations and a finite-difference staggered nonuniform grid. The rectangular domain and the nodes located on the lines parallel to the boundaries are considered. This scheme mimics the nontrivial characteristics of underlying problem that admits an existence of continuous family of steady states.

Our results show that the uniform grid is the best choice to compute the whole continuous family of steady states. It allows for the provision of equal conditions for all states belonging to the family. However for the chosen specific convective pattern we can arrange the mesh in a nonuniform manner to reveal the essential details with the highest possible resolution.

The mimetic discretizations developed in [12,13] deal with general quadrilateral meshes being more suitable for complex geometries. It is of great interest to extend the discretization for the Darcy convection to the case of quadrilateral meshes.

Acknowledgements

The authors acknowledge the support of NATO-CP Advanced Fellowship Programme of TÜBİTAK (Turkish Scientific Research Council). V.T. was partially supported by the Program for the leading scientific schools (# 1768.2003.1), the Program “Russian Universities” (# UR.04.01.035) and Russian Foundation for Basic Research (# 04-01-96815 and 05-01-00567).

References